Class IX Session 2025-26
Subject - Mathematics
Sample Question Paper - 8

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. The coordinates of a point A on y-axis, at a distance of 4 units from x-axis and below it, are **[1]**

   a) (0, 4) b) (-4, 0)

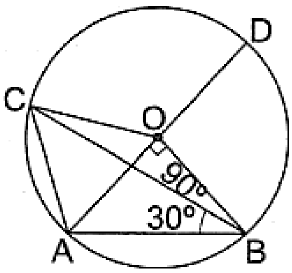
   c) (0, -4) d) (4, 0)

2. The base of a triangle is 12cm and height is 8cm then area of triangle is **[1]**

   a) $48\ cm^2$ b) $24\ cm^2$

   c) $56\ cm^2$ d) $96\ cm^2$

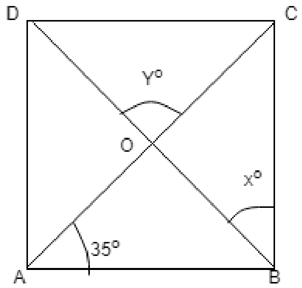
3. In the given figure, ∠AOB = 90° and ∠ABC = 30°. Then, ∠CAO = ? **[1]**



   a) 90° b) 30°

   c) 45° d) 60°

4. In the figure, ABCD is a Rectangle. Find the values of x and y? **[1]**



a) x = 50° and y = 100°

b) x = 55° and y = 110°

c) x = 100° and y = 100°

d) x = 60° and y = 120°

5. If a is rational and $\sqrt{b}$ is irrational, then $a+\sqrt{b}$ is: **[1]**

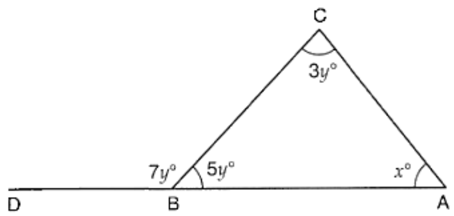
a) a rational number

b) a natural number

c) an irrational number

d) an integer

6. In figure, what is the value of x? **[1]**



a) 60

b) 45

c) 35

d) 50

7. If a linear equation has solutions (1, 2), (-1, -16) and (0, -7), then it is of the form **[1]**

a) x - 9y = 7

b) 9x - y + 7 = 0

c) y = 9x - 7

d) x = 9y - 7

8. If $x-\frac{1}{x}=\frac{15}{4}$, then $x+\frac{1}{x}=$ **[1]**

a) 4

b) $\frac{17}{4}$

c) $\frac{1}{4}$

d) $\frac{13}{4}$

9. Every rational number is **[1]**

a) a whole number

b) a real number

c) a natural number

d) an integer

10. In which of the following figures are the diagonals equal? **[1]**

a) Parallelogram

b) Rhombus

c) Trapezium

d) Rectangle

11. The value of $(32)^{\frac{1}{5}}+(-7)^{0}+(64)^{\frac{1}{2}}$ is **[1]**

a) 0

b) 10

c) 1

d) 11

12. The point on the graph of the linear equation 2x + 5y = 19, whose ordinate is 1½ times its abscissa is **[1]**

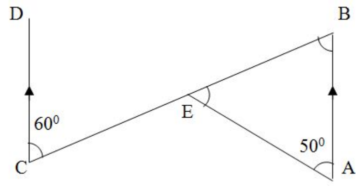
a) (2, 3)

b) (-4, -6)

c) (4, 6)

d) (-2, -3)

13. In the given figure, AB || CD. If ∠EAB = 50° and ∠ECD = 60°, then ∠AEB =? **[1]**



a) 55°

b) 70°

c) 50^0

d) 60°

14. The simplest rationalising factor of $\sqrt{3}+\sqrt{5}$,is **[1]**

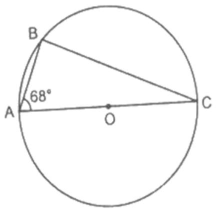
a) $\sqrt{3}+\sqrt{5}$

b) $3-\sqrt{5}$

c) $\sqrt{3}-5$

d) $\sqrt{3}-\sqrt{5}$

15. In the given figure, O s the centre of circle, $\angle BAO = 68^o$, AC is diameter of circle, then measure of $\angle BCO$ is : **[1]**



a) 33^o

b) 44^o

c) 22^o

d) 68^o

16. The point whose abscissa and ordinate have different signs will lie in **[1]**

a) II and IV quadrant

b) II and III quadrants

c) I and III quadrants

d) I and II quadrants

17. Express y in terms of x in the equation 5y - 3x - 10 = 0. **[1]**

a) $y=\frac{3x+10}{5}$

b) $y=\frac{3-10x}{5}$

c) $y=\frac{3x-10}{5}$

d) $y=\frac{3+10x}{5}$

18. If $x+\frac{1}{x}=5$, then $x^2+\frac{1}{x^2}=$ **[1]**

a) 27

b) 10

c) 23

d) 25

19. **Assertion (A):** Two opposite angles of a parallelogram are $(3x - 2)^o$ and $(50 - x)^o$. The measure of one of the angle is 37^o. **[1]**

Reason (R): Opposite angles of a parallelogram are equal.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Three rational numbers between $\frac{2}{5}$ and $\frac{3}{5}$are $\frac{9}{20}$, $\frac{10}{20}$ and $\frac{11}{20}$ **[1]**

Reason (B): A rational number between two rational numbers p and q is $\frac{1}{2}(p+q)$

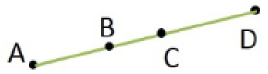
a) Both A and R are true and R is the correct

b) Both A and R are true but R is not the explanation of A. | correct explanation of A.

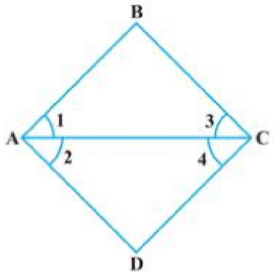
c) A is true but R is false. | d) A is false but R is true.

Section B

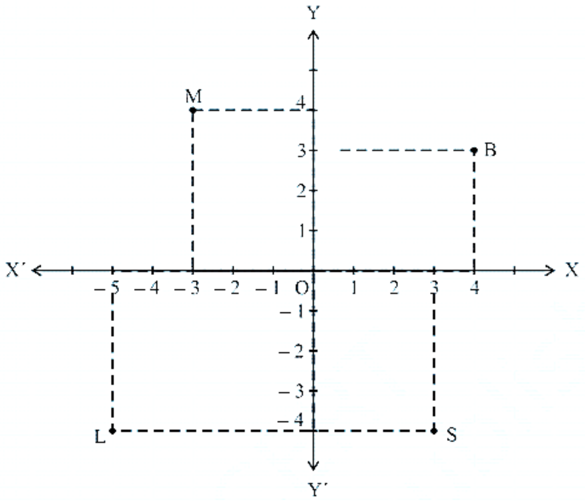
21. In fig., if AC = BD, then prove that AB = CD **[2]**



22. In the given figure, we have $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$. Show that $\angle A = \angle C$. **[2]**



23. See Fig. and complete the statement: The abscissa and the ordinate of the point B are ________ and ________, respectively. Hence, the coordinates of B are (________, ________). **[2]**



24. If $x = \frac{1}{3-\sqrt{8}}$ find the value of $x^3 - 2x^2 - 7x + 5$. **[2]**

OR

Express the decimal $0.\overline{235}$ in the form $\frac{p}{q}$, where p, q are integers and $q \neq 0$.

25. The surface area of a cuboid is 758 cm^2. Its length and breadth are 14 cm and 11 cm respectively. Find its height. **[2]**

OR

A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Section C

26. Rationalise the denominator: $\frac{1}{\sqrt{7}+\sqrt{6}-\sqrt{13}}$ **[3]**

27. If $\triangle$ABC is an isosceles triangle such that AB = AC and AD is an altitude from A on BC. Prove that **[3]**

i. $\angle B = \angle C$

ii. AD bisects BC

iii. AD bisects $\angle A$.

28. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD such that $AC \perp BD$. Prove that PQRS is a rectangle. **[3]**

29. Draw the graph of the following equation and check whether : **[3]**

i. x = 2, y = 5

ii. x = –1, y = 3 are the solutions : 5x + 3y = 4

30. ABC is an isosceles triangle in which AC = BC. AD and BE are respectively two altitudes to sides BC and AC. Prove that AE = BD. **[3]**

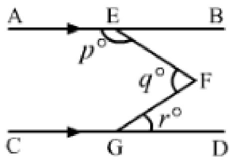
OR

ABCD is a square. X and Y are points on sides AD and BC respectively such that AY = BX. Prove that BY = AX and ∠BAY = ∠ABX.

31. Factorise: $x^3 + x^2 - 4x - 4$ **[3]**

Section D

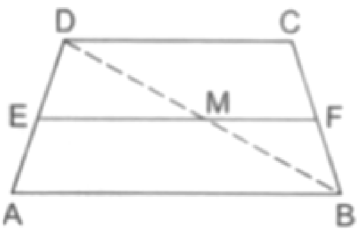
32. In the given figure, AB || CD. Prove that p + q - r = 180. **[5]**



OR

If two parallel lines are intersected by a transversal, then prove that the bisectors of the interior angles form a rectangle.

33. In Fig., ABCD is a trapezium in which side AB is parallel to side DC and E is the mid-point of side AD. If F is a point on the side BC such that the segment EF is parallel to side DC. Prove that F is the mid-point of BC and EF $= \frac{1}{2}$ (AB + DC). **[5]**



34. Find the area of the given trapezium PQRS in which RQ || SP and PQ ⊥ SP such that RQ = 7 m, RS = 13 m and SP = 12 m. **[5]**

OR

Find the area of the quadrilateral ABCD in which AB = 9 m, BC = 40 m, $\angle ABC = 90^\circ$, CD = 15 m and AD = 28 m.

35. Verify that $x^3 + y^3 + z^3 - 3xyz$ **[5]**

$$= \frac{1}{2}(x + y + z)\left[(x - y)^2 + (y - z)^2 + (z - x)^2\right]$$

Section E

36. Basant Kumar is a farmer. He has an agricultural field in the form of a rectangle of length 20 m and width 14 m. He planned to prepare compost manure in his field. For this purpose, a pit 6 m long, 3 m wide and 2.5 m deep is dug in a corner of the field and the earth taken out of the pit is spread uniformly over the remaining area of the field. **[4]**

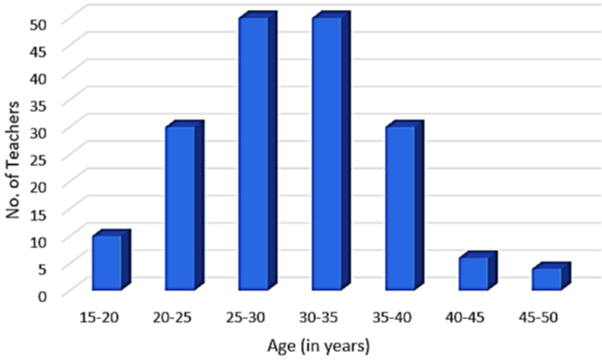
i. Find the area of the remaining part of the field.

ii. Find the extent to which the level of the field has been raised.

37. **Read the following text carefully and answer the questions that follow:** **[4]**

A teacher is a person whose professional activity involves planning, organizing, and conducting group activities to develop student's knowledge, skills, and attitudes as stipulated by educational programs. Teachers may work with students as a whole class, in small groups or one-to-one, inside or outside regular classrooms. In this indicator, teachers are compared by their average age and work experience measured in years.

For the same in 2015, the following distribution of ages (in years) of primary school teachers in a district was collected to evaluate the teacher on the above-mentioned criterion.



i. What is the total no of teachers? (1)

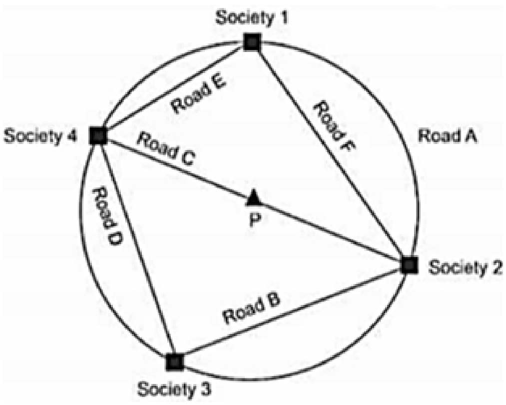
ii. Find the class mark of class 15 - 20, 25 - 30 and 45 - 50? (1)

iii. What is the no of teachers of age range 25 - 40 years? (2)

OR

Which classes are having same no. of teachers? (2)

38. Two new roads, Road E and Road F were constructed between society 4 and 1 and society 1 and 2. **[4]**



i. What would be the measure of the sum of angles formed by the straight roads at Society 1 and society 3?

a. 60^o

b. 90^o

c. 180^o

d. 360^o

ii. Krish says, The distance to go from society 4 to society 2 using Road D will be longer that the distance using Road E. Is Krish correct? Justify your answer with examples.

iii. Road G, perpendicular to Road F was constructed to connect the park and Road F. Which of the following is true for Road G and Road F?

a. Road G and road F are of same length.

b. Road F divides Road G into two equal parts.

c. Road G divides Road F into two equal parts.

d. The length of road G is one-fourth of the length of Road F.

iv. Priya said, Minor arc corresponding to Road B is congruent to minor arc corresponding to Road D. Do you agree with Priya? Give reason to supportyour answer.

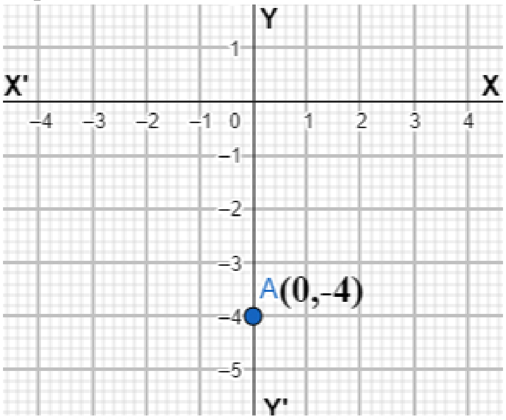
Solution

Section A

1.

(c) (0, -4)

Explanation:



∴ Coordinate of A(0, -4)

2. **(a)** 48 cm^2

Explanation:

Area of triangle = $\frac{1}{2} \times base \times height$

$= \frac{1}{2} \times 12 \times 8 = 48$

3.

(d) 60°

Explanation:

We have:

∠AOB = 2∠ACB

$\Rightarrow \angle \mathrm{ACB} = \frac{1}{2} \angle \mathrm{AOB} = \left(\frac{1}{2} \times 90^{\circ}\right) = 45^{\circ}$ => ∠ACB = 45°

∠COA = 2∠CBA = (2 × 30°) = 60°

∴ ∠COD = 180° - ∠COA = (180° - 60°) = 120°

$\Rightarrow \angle \mathrm{CAO} = \frac{1}{2} \angle \mathrm{COD} = \left(\frac{1}{2} \times 120^{\circ}\right) = 60^{\circ}$ => ∠CAO = 60°

4.

(b) x = 55° and y = 110°

Explanation:

ABCD is a rectangle

The diagonals of a rectangle are congruent and bisect each other. Therefore, in △AOB, we have:

OA = OB

∠OAB = ∠OBA = 35°

x = 90° – 35° = 55° and ∠AOB = 180° – (35° + 35°) = 110°

y = ∠AOB = 110° [Vertically opposite angles]

Hence, x = 55° and y = 110°

5.

(c) an irrational number

Explanation:

Let a be rational and $\sqrt{b}$ is irrational.

If possible let $a + \sqrt{b}$ be rational.

Then $a+\sqrt{b}$ is rational and a is rational.
$\Rightarrow \left[\left(a+\sqrt{b}\right)-a\right]$ is rational [Difference of two rationals is rational]
$\Rightarrow \sqrt{b}$ is rational.
This contradicts the fact that $\sqrt{b}$ is irrational.
The contradiction arises by assuming that $a+\sqrt{b}$ is rational.
Therefore, $a+\sqrt{b}$ is irrational.

6. **(a)** 60
Explanation:
In $\triangle$ABC,
$\angle$BCA + $\angle$CAB + $\angle$ABC = 180°
$\Rightarrow 3y^\circ + x^0 + 5y^\circ$ = 180°
$\Rightarrow 8y^\circ + x^\circ$ = 180° ....(i)
Also, $5y^\circ$ = 180° - $7y^\circ$
$\Rightarrow 12y^\circ = 180^\circ$
$\Rightarrow y^\circ = 15^\circ$
From (i), $x^0 = 180^\circ - 8y^\circ$
$\Rightarrow x^0 = 180^\circ - 8 \times 15^\circ$
$\Rightarrow x^0 = 60^\circ$

7.
(c) y = 9x - 7
Explanation:
Since all the given co- ordinate (1, 2), (-1, -16) and (0, -7) satisfy the given line y = 9x - 7
For point (1, 2)
y = 9x - 7
2 = 9(1) - 7
2 = 9 - 7
2 = 2
Hence (2, 1) is a solution.
For point (-1, -16)
y = 9x - 7
-16 = 9(-1) - 7
-16 = -9 - 7
-16 = -16
Hence (-1, -16) is a solution.
For point (0,-7)
y = 9x - 7
-7 = 9(0) -7
-7 = -7
Hence (0, -7) is a solution.

8.
(b) $\frac{17}{4}$
Explanation:
$\Rightarrow x - \frac{1}{x} = \frac{15}{4}$
Now , $\left(x - \frac{1}{x}\right)^2 = \left(\frac{15}{4}\right)^2$
$\Rightarrow (x^2) + \left(\frac{1}{x^2}\right) - 2\times x \times \frac{1}{x} = \frac{225}{16}$
$\Rightarrow (x^2) + \left(\frac{1}{x^2}\right) = \frac{225}{16} + 2$
$\Rightarrow (x^2) + \left(\frac{1}{x^2}\right) = \frac{257}{16}$
$\Rightarrow (x^2) + \left(\frac{1}{x^2}\right) + 2\times x \times \frac{1}{x} = \frac{257}{16} + 2\times x \times \frac{1}{x}$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 = \frac{257+32}{16} = \frac{289}{16}$
$\Rightarrow \left(x+\frac{1}{x}\right) = \sqrt{\frac{289}{16}} = \frac{17}{4}$

9.

(b) a real number
Explanation:
Every rational number (1, 4.5, 10, 1/2, -27, 75/5, 0) is a real number.
However, not every real number , is a rational number.
Although some numbers that appear to be irrational are actually rational because they can be reduced i.e. √25 = 5

10.

(d) Rectangle
Explanation:
Rectangle is the correct answer. As we know that from all the quadrilaterals given in other options, diagonals of a rectangle are equal.

11.

(d) 11
Explanation:
$(32)^{\frac{1}{5}} + (-7)^0 + (64)^{\frac{1}{2}}$
= 2 + 1 + 8
= 11

12. **(a)** (2, 3)
Explanation:
Ordinate means y-coordinate. It means we need to find a point on the given line where y-coordinte = 3/2 (x-coordinate) . Just put y = [(3/2) .x] in the given eqn.
$2x + 5 \cdot \frac{3}{2}x = 19$
$2x + \frac{15}{2}x = 19$
$\frac{4x+15x}{2} = 19$
$\frac{19x}{2} = 19$
$x = \frac{19\times 2}{19}$
$y = \frac{3}{2}x$
$y = \frac{3}{2} \times 2$
y=3
so the co-ordinate are (2,3)

13.

(b) 70°
Explanation:
∠BCD =∠ ABE = 60° (Vertically opposite angle)
In △EAB
∠EAB + ∠EBA + ∠AEB + 180^0 (Angle sum property)
50° + 60° + ∠AEB = 180°
∠AEB = 70°

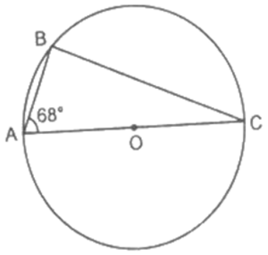
14.

(d) $\sqrt{3} - \sqrt{5}$
Explanation:
The simplest rationalising factor of $\sqrt{3} + \sqrt{5}$ is $\sqrt{3} - \sqrt{5}$

15.

(c) 22^o

Explanation:



$\angle B = 90^o$ (Angle in a semicircle)

Now, in $\triangle ABC$

$\angle A + \angle B + \angle C = 180^o$

$68^o + 90^o + \angle C = 180^o$

16. **(a)** II and IV quadrant

Explanation:

A coordinate is an ordered pair of numbers in which the first number is the abscissa i.e. the measurement along the X axis.

The signs of the coordinates in the first quadrant is (+,+),

the second quadrant is (−,+),

the third quadrant is (−,−),

fourth quadrant is (+,−).

So, the points whose abscissa and ordinate have different signs are 2nd and 4th quadrants.

17. **(a)** $y = \frac{3x+10}{5}$

Explanation:

5y - 3x - 10 = 0

5y - 3x = 10

5y = 10 + 3x

y = $\frac{10+3x}{5}$

18.

(c) 23

Explanation:

Using ,$(a + b)^2 = a^2 + b^2 + 2ab$

$\left(x + \frac{1}{x}\right)^2 = x^2 + \left(\frac{1}{x^2}\right) + 2x\frac{1}{x}$

$\Rightarrow (5)^2 = x^2 + \left(\frac{1}{x^2}\right) + 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 25 - 2$

$x^2 + \frac{1}{x^2} = 23$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Since, opposite angles of a parallelogram are equal.

Therefore, 3x - 2 = 50 - x

x = 13

One angle is 37^o

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

Section B

21. AC = BD . . . . [Given] . . . (1)

AC = AB + BC . . . . [Point B lies between A and C] . . . . (2)

BD = BC + CD . . . . [Point C lies between B and D] . . . . (3)
Substituting (2) and (3) in (1), we get
AB + BC = BC + CD
$\Rightarrow$ AB = CD . . . . [Subtracting equals from equals]

22. We have $\angle 1 = \angle 3$ ...(1) [Given]
And $\angle 2 = \angle 4$ ...(2) [Given]
Now, by Euclid's axiom 2, we have if equal are added to equals, the whole are equal.
Adding (1) and (2), we get
$\angle 1 + \angle 2 = \angle 3 + \angle 4$
Hence, $\angle$A = $\angle$C.

23. Since the distance of point B from the y-axis is 4 units. Thus x - coordinate or abscissa of point B is 4. The distance of point B from the x-axis is 3 units. Therefore, y - coordinate or ordinate of point B is 3.
Hence, the coordinates of point B are (4, 3).

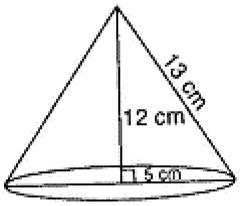
24. We have
x = $\frac{1}{3-\sqrt{8}}$
= $\frac{1}{(3-\sqrt{8})} \times \frac{(3+\sqrt{8})}{(3+\sqrt{8})}$
= $\frac{(3+\sqrt{8})}{(3)^2-(\sqrt{8})^2}$
= $\frac{(3+\sqrt{8})}{9-8}$
= $(3 + \sqrt{8})$
hence x = 3 +$\sqrt{8}$ $\Rightarrow$ x - 3 =$\sqrt{8}$
$\Rightarrow$ (x - 3)^2 = ($\sqrt{8}$)^2
$\Rightarrow$ x^2 + 9 - 6x = 8
$\Rightarrow$ x^2 - 6x + 1 = 0
$\therefore$ x^3 - 2x^2 - 7x + 5
= x(x^2 - 6x + 1) + 4(x^2 - 6x + 1) + 16x + 1
= x $\times$ 0 + 4 $\times$ 0 + 16(3 +$\sqrt{8}$) + 1
= 48 + 16$\sqrt{8}$ + 1
= 49 + 32$\sqrt{2}$

OR

Let x = $0.\overline{235}$
i.e. x = 0.235235..... ....(i)
Multiply both sides by 1000, we get
$\Rightarrow$ 1000x = 235.235235....... ....(ii)
On subtracting (i) from (ii), we get
999x = 235
$\Rightarrow x = \frac{235}{999}$
$\therefore 0.\overline{235} = \frac{235}{999}$

25. Surface area of cuboid = 758 cm^2
Length of cuboid = 14 cm
Breadth of cuboid = 11 cm
Let height of cuboid = h cm
Total surface area of cuboid = 2 (lb + bh + hl)
$\Rightarrow$ 758 = 2 (14 $\times$ 11 + 11h + 14h)
$\Rightarrow$ 154 + 25h $= \frac{758}{2} = 379$
$\Rightarrow$ 25h = 379 – 154 = 225
$\Rightarrow$ h = $\frac{225}{25} = 9$
Height of cuboid = 9 meter

OR



The solid obtained will be a right circular cone whose radius of the base is 5 cm. and height is 12 cm

$\therefore$ r = 5 cm, h = 12 cm

$\therefore$ Volume = $\frac{1}{3}\pi r^2 h$

$\frac{1}{3} \times \pi \times (5)^2 \times 12\ cm^3$

= 100π cm^3

The volume of the solid so obtained is 100π cm^3

Section C

26. $\frac{1}{\sqrt{7}+\sqrt{6}-\sqrt{13}}$

$\frac{1}{(\sqrt{7}+\sqrt{6})-\sqrt{13}} \times \frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{(\sqrt{7}+\sqrt{6})+\sqrt{13}}$

$= \frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{(\sqrt{7}+\sqrt{6})^2-\sqrt{13}^2}$ [$\because$a^2 - b^2 = (a + b)(a - b)]

$= \frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{(7+6+2\sqrt{42})-13}$

$= \frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{13+2\sqrt{42}-13}$

$= \frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{2\sqrt{42}}$

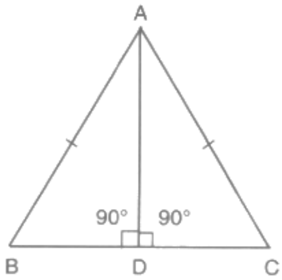
$= \frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{2\sqrt{42}} \times \frac{\sqrt{42}}{\sqrt{42}}$

$= \frac{\sqrt{7\times42}+\sqrt{6\times42}+\sqrt{13\times42}}{2(\sqrt{42})^2}$

$= \frac{\sqrt{7\times7\times6}+\sqrt{6\times6\times7}+\sqrt{546}}{2\times42}$

$= \frac{7\sqrt{6}+6\sqrt{7}+\sqrt{546}}{84}$

27. In right triangles ADB and ADC, we have



According to question Hyp. AB = Hyp. AC [Given]

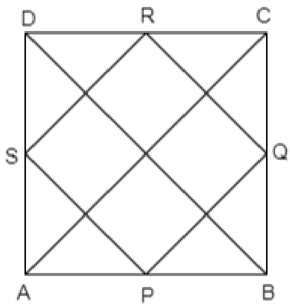
AD = AD [Common side]

So, by RHS criterion of congruence, we have

$\Delta ABD \cong \Delta ACD$

$\Rightarrow \angle B = \angle C$, BD = DC and $\angle BAD = \angle CAD$ [c.p.c.t.]

$\Rightarrow$ AD bisect BC and $\angle A$

28. 

Given, P, Q, R and S are mid-points of the sides AB, BC, CD and DA, respectively.

Also,

AC is perpendicular to BD

$\therefore\angle$COD = $\angle$AOD = $\angle$AOB = $\angle$COB = 90°

In △ADC, by mid-point theorem,

SR || AC and SR = $\frac{1}{2}$AC

In △ABC, by mid-point theorem,

PQ || AC and PQ = $\frac{1}{2}$AC

∴ PQ || SR and SR = PQ = $\frac{1}{2}$AC

Similarly,

SP || RQ and SP = RQ = $\frac{1}{2}$BD

Now, in quad EOFR,

OE || FR, OF || ER

∠EOF = ∠ERF = 90°

Hence, PQRS is a rectangle.

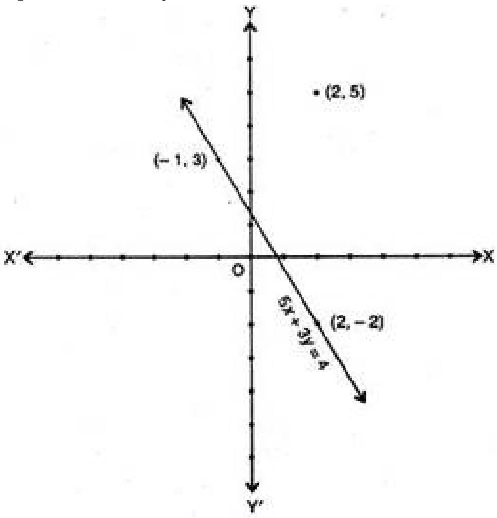
29. 5x + 3y = 4

⇒ 3y = 4– 5x

⇒ y = $\frac{4-5x}{3}$

x	2	–1
y	–2	3

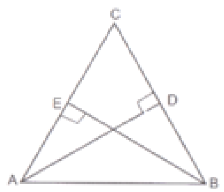
We plot the points (2, –2) and (–1, 3) on the graph paper and join the same by a ruler to get the line which is the graph of the equation 5x + 3y = 4



i. ∵The point (2, 5) does not lie on the graph
   ∴ x = 2, y = 5 is not a solution.

ii. ∵The point (–1, 3) lies on the graph
   ∴ x = –1, y = 3 is a solution.

30. In ΔADC and ΔBEC we have



AC = BC [Given] …(1)

$\angle ADC = \angle BEC$ [Each = 90°]

$\angle ACD = \angle BCE$ [Common angle]

∴ $\Delta ADC \cong \Delta BEC$ [By SSS congruence rule]

∴ $CE = CD$ …(2) [CPCT]

Subtracting (2) from (1), we get

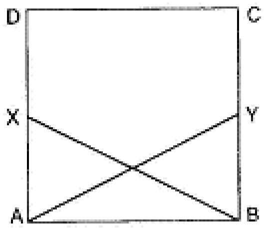
AC - CE = BC - CD

⇒ AE = BD

Hence, proved.

OR

In right triangles △BAY and △ABX,



AY = BX . . . [Given]

side AB = side AB . . . [Common]

∠ABY = ∠BAX . . . [Each 90]

∴ △BAY ≅△ABX. . .[R.H.S. axiom]

∴ BY = AX . . . [c.p.c.t]

and ∠BAY = ∠ABX . . . .[c.p.c.t.]

31. Let f(x) = $x^3 + x^2 - 4x - 4$ be the given polynomial.

We have, f(-1) = $(-1)^3 + (-1)^2 - 4(-1) - 4 = -1 + 1 + 4 - 4 = 0$

And, f(2) = $(2)^3 + (2)^2 - 4(2) - 4 = 8 + 4 - 8 - 4 = 0$

So, (x + 1) and (x - 2) are factors of f(x).

⇒ (x + 1)(x - 2) is also a factor of f(x).

⇒ $x^2 - x - 2$ is a factor of f(x).

Let us know divide f(x) = $x^3 + x^2 - 4x - 4$ by $x^2 - x - 2$ to get the other factors of f(x).

By long division, we have

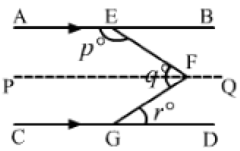
$$x^2 - x - 2 \overline{)\, x^3 + x^2 - 4x - 4 \,(}\; x + 2$$
$$\underset{-}{x^3} \underset{+}{- x^2} \underset{+}{+ 2x}$$
$$2x^2 - 2x - 4$$
$$\underset{-}{2x^2} \underset{+}{- 2x} \underset{+}{- 4}$$
$$0$$

∴ $x^3 + x^2 - 4x - 4 = (x^2 - x - 2)(x + 2)$

⇒ $x^3 + x^2 - 4x - 4 = (x + 1)(x - 2)(x + 2)$

Hence, $x^3 + x^2 - 4x - 4 = (x - 2)(x + 1)(x + 2)$

Section D

32. 

Draw PFQ || AB || CD

Now, PFQ || AB and EF is the transversal.

Then,

$\angle AEF + \angle EFP = 180^\circ$ ...(i)

[Angles on the same side of a transversal line are supplementary]

Also, PFQ || CD.

$\angle PFG = \angle FGD = r^\circ$ [Alternate Angles]

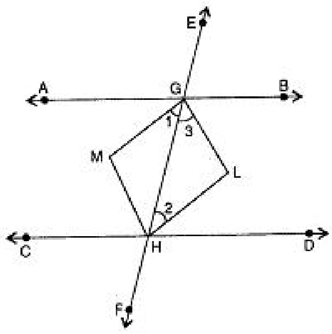
and $\angle EFP = \angle EFG - \angle PFG = q^\circ - r^\circ$

putting the value of ∠EFP in equation (i)

we get,

p° + q °- r° = 180° [∠AEF = p°]

OR



as, AB || CD and EF cuts them

∴ ∠AGH = ∠GHD (Alternate Angles)

⇒ $\frac{1}{2}\angle AGH = \frac{1}{2}\angle GHD$

⇒ ∠1 = ∠2 .... (1)

But these angles form a pair of equal alternate angles for lines GM and HL and transversal GH.

∴ GM || HL .... (2)

Similarly, we can prove that

HM || GL .... (3)

In view of (2) and (3),

GLHM is a parallelogram

AB || CD and EF cuts them

∴ ∠BGH + ∠GHD = 180^o

(The sum of the interior angles on the same side of a transversal is 180^o)

⇒ $\frac{1}{2}\angle BGH + \frac{1}{2}\angle GHD = 90^o$

⇒ ∠3 + ∠2 = 90^o

In △GHL,

∠3 + ∠2 + ∠GLH = 180^o

(The sum of the three angles of a triangle is 180^o)

⇒ 90^o + ∠GLH = 180^o .... From (4)

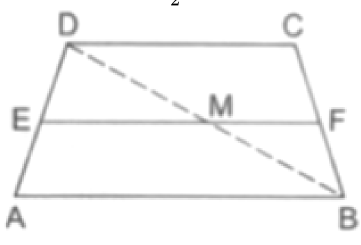
⇒ ∠GLH = $180^o - 90^o = 90^o$

⇒ One angle of parallelogram GLHM is a right angle.

⇒ Parallelogram GLHM is a rectangle.

33. A trapezium ABCD in which AB || DC, E is the mid-point of AD and F is a point on BC such that EF || DC.

To Prove: EF = $\frac{1}{2}$ (AB + DC)



PROOF: In △ADC, E is the mid-point of AD and EM || DC (Given)

∴ G is the mid-point of AC

Since segment joining the mid-points of two sides of a triangle is half of the third side.

∴ EM = $\frac{1}{2}$DC ...(i)

Now, ABCD is a trapezium in which AB || DC

But, EF || DC

∴ EF || AB

⇒ MF || AB

In △ABC, M is the mid-point of AC (proved above) and MF || AB.

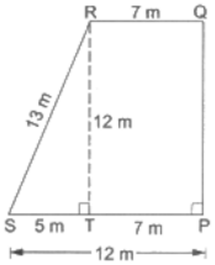
∴ F is the mid-point of BC

⇒ MF = $\frac{1}{2}$AB [∵ Segment joining the mid-points of two sides of a △ is half of the third sides] ...(ii)

From (i) and (ii), we have

EM + MF = $\frac{1}{2}$ (DC) + $\frac{1}{2}$(AB)

$\Rightarrow$ EF = $\frac{1}{2}$ (AB + DC).

34. 

Draw RT $\perp$ SP. Then, TP = RQ - 7m .

$\therefore$ ST = SP - TP = (12 - 7) m = 5 m.

From right $\triangle$RTS, we have

$RT^2 = RS^2 - ST^2 = [(13)^2 - (5)^2]m^2$

$RT^2 = (169 - 25)\ m^2 = 144\ m^2$.

$\therefore RT = \sqrt{144}\text{m}$ = 12 m

Area of trapezium PQRS

$= (\frac{1}{2} \times$ sum of parallel sides $\times$ distance between them)

$= \{\frac{1}{2} \times (RQ + SP) \times RT\}$ sq units

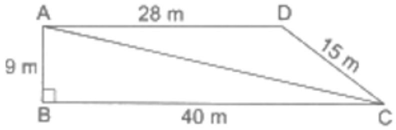
$= \{\frac{1}{2} \times (7 + 12) \times 12\}\ \text{m}^2$

$= (19 \times 6)\text{m}^2$

Area of trapezium PQRS= 114 m^2

Hence, the area of the trapezium PQRS is 114 m^2

OR



Area of right $\triangle$ABC

$= \left(\frac{1}{2} \times \text{ base } \times \text{ height}\right)$

$= \left(\frac{1}{2} \times 40 \times 9\right)\text{m}^2$ = 180 m^2

Also, from right $\triangle$ABC, we get

$AC^2 = AB^2 + BC^2$ [using paythagoras theorem]

$\Rightarrow AC^2 = [(9)^2 + (40)^2]\ m^2$

$\Rightarrow AC^2 = (81 + 1600)\ m^2 = 1681\ m^2$

$\Rightarrow AC = \sqrt{1681}\text{m}$ = 41 m

In $\triangle$ACD, we have AC = 41 m, CD = 15 m and AD = 28 m.

Let a = 41 m, b = 15 m and c = 28 m. Then,

s = $\frac{1}{2}$ (41 + 15 + 28)m $= \left(\frac{1}{2} \times 84\right) m$ = 42 m

$\therefore$ (s - a) = (42 - 41)m = 1 m, (s - b) = (42 - 15)m = 27 m

and (s - c) = (42 - 28) m = 14 m

$\therefore$ area of $\triangle$ACD $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42 \times 1 \times 27 \times 14}\text{m}^2$

$= (14 \times 9)\text{m}^2$

= 126 m^2

Area of the quadrilaterals ABCD = area ($\triangle$ABC) + area ($\triangle$ACD)

= (180 + 126) m^2 = 306 m^2

35. LHS is $x^3 + y^3 + z^3 - 3xyz$

and RHS is $\frac{1}{2}(x+y+z)\left[(x-y)^2 + (y-z)^2 + (z-x)^2\right]$.

We know that $x^3 + y^3 + z^3 - 3xyz = (x+y+z)\left(x^2 + y^2 + z^2 - xy - yz - zx\right)$ ....(i)

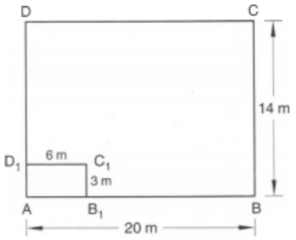
And also, we know that $(x-y)^2 = x^2 - 2xy + y^2$ ....(ii)

R.H.S. = $\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$
$\frac{1}{2}(x+y+z)\left[\left(x^2-2xy+y^2\right)+\left(y^2-2yz+z^2\right)+\left(z^2-2xz+x^2\right)\right]$ [From eq.(i) and (ii)]
$\frac{1}{2}(x+y+z)\left(2x^2+2y^2+2z^2-2xy-2yz-2zx\right)$
$(x+y+z)\left(x^2+y^2+z^2-xy-yz-zx\right)$.
Therefore, we can conclude that the desired result is verified

Section E

36. Let ABCD be the field and let $AB_1C_1D_1$ be the part of the field where a pit is to be dug.

Volume of the earth dug out = $(6 \times 3 \times 2.5)$ m^3 = 45 m^3 ...(i)
Area of the remaining part of the field = Area of the field - Area of pit
= $(20 \times 14 - 6 \times 3)$m^2 = 262m^2
The earth taken out of the pit is spread uniformly over the remaining area of the field. Let h metres be the level raised over the field uniformly. Clearly, the earth taken out forms a cuboid of base area 262 m^2 and height h.
Volume of the earth dug out = $(262 \times h)$ m^3 ......(ii)



From (i) and (ii), we have
262h = 45 $\Rightarrow$ h = $\frac{45}{262}$ = 0.1718m = 17.18cm
Hence, the level is raised by 17.18 cm

37. i. No of teachers in the age-group 15-20 years = 10
No of teachers in the age-group 20-25 years = 30
No of teachers in the age-group 25-30 years = 50
No of teachers in the age-group 30-35 years = 50
No of teachers in the age-group 35-40 years = 30
No of teachers in the age-group 40-45 years = 5
No of teachers in the age-group 45-50 years = 2
Thus the total no of teachers
= 10 + 30 + 50 + 50 + 30 + 5 + 2
=177

ii. Class Mark of class 15 - 20 =
$= \frac{15+20}{2}$ = 17.5
Class Mark of class 25 - 30 =
$= \frac{25+30}{2}$ = 27.5
Class Mark of class 45 - 50 =
$= \frac{45+50}{2}$ = 47.5

iii. No of teachers in the age-group 25 - 30 years = 50
No of teachers in the age-group 30 - 35 years = 50
No of teachers in the age-group 35 - 40 years = 30
Thus the no of teachers in the age range 25 - 40 years
= 50 + 50 + 30 = 130
OR
From the observation of the bar chart we find that :
No of teachers in the age-group 25-30 years = 50
No of teachers in the age-group 30-35 years = 50
Thus the no of teacher in the class 25-30 and 30-35 is equal.

38. i. (c) 180^{o}

ii. Show that in a right triangle the sum of legs is longest for an isosceles right triangle when hypotenuse remains same.
Take for example the length of diameter (hypotenuse) = 5 units.
Road D and Road B are equal hence (Road D = 3.53 units).
Let Road E be = 1, Road F = 4.89 units.
Therefore, length of Road B + Road D is greater than Road E + Road F.
iii. (c) Road G divides Road F into two equal.
iv. Yes, Priya is correct because arc corresponding to two equal roads (chords) are congruent.